Mathematical Modelling in Economics

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Abstract

This study examines the significance of fundamental and advance mathematics in theoretical as well as real world economic analysis.

It focuses on use of different mathematical operational tools and techniques to solve economic problems. The main significance of mathematical Economics is that it helps to solve the Optimisation problems like profit and output maximization given the labour and capital constraints and cost minimization. It is the main help to managerial economists and business analysts for the betterment of the firm.

The framework of Economic model can be mathematical and non-mathematical in nature. The model will consist of set of equations structured to explain the model. The relation of variables in certain ways helps these equations to give mathematical form to the analytical assumptions used.

Though mathematics has become advanced mode of communication in economic analysis as it helps to quantify the relation between economic variables but still there is criticism for the application of mathematical modelling in economics because sometimes textbook models fail to take in account real world scenario.

Keywords: *communication, economics, managerial mathematics*

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Introduction

Mathematical tools such as calculus functions of one or more variables, single and multivariable optimisation problems,with and without constraints, linear algebra are helpful for economics researchers, students and professors in various branches of Economics including, labour economics, industrial organisation and public finance. The concept of differentiation and differential equationare used frequently in areas of Economics such as development and environment economics are used frequently.

From ancient times, economic activity has been playing significant part of human life. There were traders and merchants who already had an understanding of economic phenomena. The most basic concepts were expressed in elementary mathematics. Mathematical operational tools like addition, subtraction, multiplication and division along with fraction and integers were sufficient to allow economic agents to trade and earn living. These mathematical tools were sufficient for all economic agents to maintain accounts and to work out what price to be charged for product or service.

Arithmetic could carry out interest calculation on loan which was not very complicated even in the times when the concept of zero and decimal system was not introduced. The reason for usage of mathematics in economic analysis is to help students obtain mathematical skills they require to read technical economic literature and also to obtain respected position as economist or business analyst in life. Economic analysis alsobecomes understandable and interesting even with the knowledge of rudimentary mathematics.

Mathematical Economics is not a clear cut branch of economics rather it is an approach for economic analysis in which an analyst make use of mathematical symbols in the statement of problem and also draw upon known mathematical theorems to be used for reasoning.

Literature Review

A significant topic in many scientific disciplines including economics is how dependent variable changes due to change in independent variable or how quantities change over the period of time. To forecast future demand for a commodity, to know the future planet position or to project the growth in population, information about rate of change is required.

Differentiation

Derivative is the central concept in mathematical analysis which is used to describe the rate of change of a function. Differential calculus has been the foundation for the development of modern science. It has also been of Central importance to the theoretical development of modern economics (Knut Sydsaeter, Peter Hammond and Arne storm 2012).

For example for an estimated cost function C=65x+52 for I-land steel Corporation Limited (C is the cost in dollars per year and x is the production of Steel in quintals per year).

The slope, $\frac{dc}{dx}$=65 means if production increases by 1 quintal, then cost increases by $65

Similarly, for another estimated annual demand function q=-0.24p+0.15for wheat (p is the price in rupees and q is the quantity per person).

The slope, $\frac{dq}{dp}$=-0.24 means that if price increases by one rupee then quantity demanded decreases by 0.24 units.

Implicit Differentiation

Implicit differentiation is more important than many other mathematical techniques applied in economics. The reason for that is, many functions are defined implicitly by a system of equation in economic models.

Example: Consider the standard macroeconomic model for calculating national income in a closed economy assuming the following:

(a)Y=C+I (b)C=f(Y)

Where (a) tells us that income is divided between consumption and investment and (b) defines consumption as a function of income ,where, f'(Y)is marginal propensity to consume and it lies between 0 and 1.

1. Suppose that C=f(Y)= 96.08+0.612Y, use equation (a) and (b) to find 'Y' in terms of 'I'

2. Putting the expression for 'C'from (b) into (a) results in Y=f(Y)+I. In this equation 'Y' is defined as differentiable function of 'I'. Solve for .

Solution:

1. Here $Y=96.08+0.612Y+I$, solving for Y, it gives

$Y=[96.08+I]/[1-0.612]= 247.63+2.58I$ which means $\frac{dY}{dI}=2.58$ so if I increases by 1 billion then national income increases by 2.58 billion dollars.

2. Differentiating $Y= f(Y)+ I$ with respect to I, it gives

$\frac{dY}{dI}= f'(Y)\frac{dY}{dI}+1$ which means $\frac{dY}{dI} =1/[1-f'(Y)]$

If $f'(Y) =0.75$ then $\frac{dY}{dI}= 4$ and for $f'(Y)=0.612$ then $\frac{dY}{dI}=2.58$.

Since f'(Y) lies lies between 0 and 1 hence 1 -f'(Y) also lies between 0 and 1 therefore 1/[1 -f'(Y)] is always greater than 1. Therefore in this case, $1billion increase in investment result in more than $1billion increase in national income

Optimisation

Optimisation is choosing the prime or the foremost of all the elements from the set of available options. Optimisation is a mathematical technique used in Economics to solve for one or more variables and through this technique firms either minimise the cost or maximise the profit and output. The management of the firm employs the combination of labour and capital (assuming two variable case) so that firm's profit is maximum and cost is minimum at that level of labour and capital(Knut Sydsaeter, Peter Hammond and Arne Storm 2012).

Here are some economic examples of optimization problem:

Suppose T(F) bushels of wheat are harvested per acre of land with 'F' pounds of fertilizers per acre are used, if 'P' is the prize in dollars per bushel of wheat and 'q' is the price in dollars per pound of fertilizers then, profits per acre in dollar are

$\pi(F)=PT(F)-qF$, for F positive, where π (profit) is function of F.

Let us assume F* maximizes profit which means if there exist F*, then Profit'(F) is positive for 'F' less than equal to F* and vice versa. Therefore $Profit'(F^*)= 0$ that is $PT'(F^*)-q=0$

And $PT'(F^*)= q$.....(#)

The economic interpretation of this is given by

If F * is increased by 1 unit then T'(F*)= T(F*+1)-T(F*) units more bushels are produced and for each bushels the study gets P dollars. Therefore, increase in gain when F* increases by 1 unit is given by PT'(F*) dollars and on the other hand the study loses q dollars because this is the cost of one unit of fertilizer

Hence……. (#) can be interpreted as:

For maximization of profits, fertilizers should amount to the level F* at which additional cost of fertilizer equates the gain it provided from the revenue of bushels.

Objectives of Study

Following are the objectives of this paper:

1. To determine the significance of mathematics in economic analysis.
2. To examine as to how the economic analysis goes with and without mathematical approach.
3. To see as to how mathematical approach helps to quantify the changes in qualitative economic variables.
4. To check as to how mathematical approach in economic analysis help to solve business related problems and gives the best optimal solutions.
5. To go beyond geometric methods while using mathematical approach in economic analysis.
6. To find out the relation between different Macro Economic parameters (GDP and Consumption rate) using regression analysis.
7. To verify the equilibrium of a particular commodity in a market using linear equation and then through application of comparative static analysis to same when the parameters are unknown.

Methodology

Economics as a social science strives to describe how the economy runs and to make forecast, how the economic variables are going to react when certain changes happen

Effect on crop prices during crop failure , effect on prices of finished good due to change in sales tax, effect of increase in government expenditure on employment are examples. Economics also helps in the efficient allocation of resources through various problem solving techniques in which mathematics is fundamental to apply to the various allocation problems of economics.

(1) **Quantification:** In economic analysis forecast are made with the help of diagram. For example, in a competitive market if supply is increased then the price of a good falls. However, there is very common economic intuition behind it which any market player can analyse.

An economist, also needs to make prediction regarding the amount by which the price falls if the quantity supplied is increased by specific number. For such economic forecast, use of mathematics is required.

Sometime the economic analysis that are non mathematical in nature can also be useful for predicting the direction of expected change and the changes which are non quantitative in nature.

(2) **Simplification:** Algebraic expressions make some economic concept or relationships much easier to understand than if they were expressed in words. For example, the relationship between the price of a particular good and the quantity of the good can be expressed as, the quantity demanded in a given time is 1,000 kg when price is zero and decreases by 20kg with increase in price for every one unit. If this was to express mathematically then it can be given by $q=1,000-20p$, where 'q' is the quantity demanded for oranges and 'p' is the price of oranges per kilogram.

The above example is very simple but when the relationship between the variables becomes complex then the significance of mathematical approach to apply in economic analysis becomes much more important.

(3) **Scarcity and Choice:** Economics deals with the optimisation problem that are concerned with the efficient allocation of limited resources. Every firm tries to maximize output given the budget constraint (expenditure on the inputs used to produce that output). Mathematics plays a key role in finding answers to such Optimisation problem. Use of mathematical techniques to solve such resource allocation problems plays a key role especially for managerial economist and operational research analyst.

Mathematical versus Non-mathematical Economics

There is no fundamental difference between mathematical and non-mathematical approach to economic analysis. The objective of any economic analysis, whether it is mathematical or non- mathematical in nature, is to obtain set of conclusion from the set of assumptions through reasoning. However, the significant differences between the two kinds of economic analysis are:

Firstly, in mathematical economic analysis assumptions and conclusions are expressed in mathematical symbols or operations and equations rather than words and sentences respectively.

Secondly, literary logic is replaced with mathematical theorems which aids to draw upon abundance of reasoning in the process.

But still symbols and word are equivalent because generally symbols are defined in words, so it does not matter much what is chosen over the other.

Mathematics forces the analyst to state assumptions explicitly at every stage just because mathematical theorems are usually stated in the 'if-then' form, so to get the then (result) part of theorem for their analysis, they should see that 'if' or conditional part does conform to the assumptions mentioned.

Looking Beyond Geometric Methods

Why is there a need to go beyond geometric methods especially while applying mathematics in economic analysis. The answer to this question is that, though geometric method has advantage of being visual, it also suffers from dimensional limitation. For example, in discussion of indifference curve the assumption is that, only two commodities are available for the consumer to consume. These assumptions are not willingly adopted but these are forced upon because the task of drawing a 3D graph is very difficult. The construction of graphs beyond three dimension is physically impossible which means to deal with cases of more than 2 goods (upto n goods), the study must resort to flexible tools of equation.

Advantages of Mathematical Approach

The mathematical approach for economic analysis serves the following purposes.

1. The language used in this approach is concise and precise;
2. There are lots of mathematical theorems which aid in reasoning for economic analysis;
3. Since, in mathematical approach the study state all the assumptions explicitly and use mathematical theorems, this keeps us away from danger of unintentional adoption of implicit assumption;
4. Moreover, mathematical approach allows us to compute for general n-variable case.

Looking at these advantages the study also hear criticism that mathematical derived theory is unrealistic. However, this criticism seems to be invalid because analysis of economic theory mathematical or non-mathematical approach cannot be unrealistic. Rather it can be said that more variables can be adopted so that the analysis can approach more towards real world solutions.

Mathematical Economics vs Econometrics

Econometrics is related to measurement of economic data. It deals with empirical observation using statistical methods to prove the relationships between economic variables. Statistical methods used mainly are estimation and hypothesis testing. On the other hand, mathematical economics is the application of mathematics to the theoretical aspects of economic analysis.

However, looking at relationship between both, empirical studies and theoretical analysis go hand in hand as they are often complimentary and mutually reinforcing.This also allows theories to be tested against empirical data for validity before their application.

Sources of Data

All the sources and text referred in this paper are mathematical economics in nature. The study has referred Essential Mathematics for Economic Analysis (4th edition 2012) by Knut Sydsaeter, Peter Hammond with Arne Storm to show how Mathematics in economic analysis helps to optimise the results and how important are mathematical techniques such as differentiation and implicit differentiation since economic variables have dependent relationship with one another and that too these are defined implicitly.The study has also used Basic Mathematics for Economist by Mike Rosser(2nd edition -2003) to define various

benefits and its significance in economic analysis. The study has also referred fundamental methods of mathematical economics (Indian edition - 4e) by Alpha C. Chaing(2013) for making comparison between mathematical and non-mathematical economics, and mathematical economics versus econometrics which apart from making comparison also defines their relationship. At various occasions Mathematics for Economic Analysis by Knut Sydsaeter and Peter J.Hammond(2006) and online sources such as 'opentextbc.ca' in which books are reviewed by Faculty of British Columbia, Canada have also been referred.

Mathematics has lot of significance in economic analysis which the study has seen through the concept of differentiation, implicit differentiation and optimisation problems and not only in one branch of economics but in many branches. Mathematics helps to explain economic model in structured and much precise way through symbols and equations. This is a much better way to define perfect or imperfect relationship between economic variables rather than defining those relationships in words and sentences.

Analysis

There are two significant ways one can state a fact or a belief in economics, one is through using experimental work or empirical evidence which is econometrics or mathematical arguments.

Table 1

Financial Year (FY_i)	Consumption*(% YoY)	Real GDP (% YoY)
FY-13	4.5	5.6
FY-14	5.8	6.6
FY-15	7.2	7.2
FY-14	6.6	7.6
FY-17	8.1	7.8
FY-18	9.0	8.1

private+governmentconsumption*, Source: Motilal Oswal Thematic June 2016,CSO,. RBI,CMIE

To examine how much real GDP is dependent upon consumption, the study is going to regress real GDP on consumption which means that real GDP is dependent variable and consumption is independent variable.

Table 2

Consumption(x_i)	Real GDP (Y_i)	x_i-m	Y_i-M	(x_i-m)(Y_i-M)	(x_i-m)(x_i-m)
4.5	5.6	-2.37	-1.55	3.674	5.6169
5.8	6.6	-1.07	-0.55	0.589	1.1449
7.2	7.2	0.33	0.05	0.017	1.1089
6.6	7.6	-0.27	0.45	-0.122	0.0729
8.1	7.8	1.23	0.65	0.799	1.5129
9.0	8.1	2.13	0.95	2.204	4.5369
Total=41.2 Mean(m)=6.87	Total=42.9 Mean(M)=7.15			$\sum[(x_i\text{-m})(Y_i\text{-M})]=6.98$	$\sum[(x_i\text{-m})(x_i\text{-m})]=12.993$

$b_2 = \sum [(x_i\text{-m})(Y_i\text{-M})] / \sum [(x_i\text{-m})(x_i\text{-m})]$

$b_2 = 6.98/12.993 = 0.537$

$b_1 = M\text{-}b_2(m)$

$b_1 = 7.15\text{-}0.537(6.87)$

$b_1 = 3.459$

Through the above found parameters (b1&b2), the study has derived the linear regression functionin which b1 is the intercept parameter and b2 is the slope coeficient. Linear regression function is given by:

$y_i = 3.459 + 0.537x_i$

This equation represents that with one unit(% YoY)change in independent variable which is consumption there is estimated 0.537(% YoY)change in real GDP given the data from FY -13 to FY-18.

Comparative static analysis is also related to empirical work or econometrics. Estimation of equation in which dependent variables are related to set of independent variable is an integral part of empirical project or work. The equation shows the dependent variable changes due to change in one of the independent variables. For example, equation is estimated for demand in which quantity demanded is the dependent variable and related goods price , it's on price and income of the consumer are independent variables, the equation may show how the quantity demanded changes when the price of related good changes. For the derivation of comparative static results, an empirical project uses maths

first and then uses data to estimate the comparative static result. It means static analysis and econometrics complement each other.

Also microeconomics is based on two important principles that is optimisation and equilibrium. The firm chooses the level of output at which profit is maximum as in the same way the consumer chooses the commodity bundle to maximize the utility level. The problems of optimisation are solved through the mathematical tool called calculus. Also optimisation problems are solved through basic as well as advanced calculus. Moving beyond basic calculus, economic agents simultaneously choose the value of more than one variable. Consumers choose the commodity bundle not the single amount of commodity. Therefore, multivariable calculus is required to analyse the problem with several variable choice.

And one of the equilibrium related problems is finding the market clearing price. And equilibrium price is the one at which quantity supplied and the quantity demanded are equal and there are no incentive for any one of the two to change. Mathematically it goes for finding the solution of equations in several unknowns. Linear algebra is the mathematical branch used here to solve system of equations.

In economics, some of the examples where linear functions are applied are following supply and demand schedules:

$$D = A - BP$$

$$S = a + bP$$

Here A&B (both positive) are parameters of demand function and in the same way a and b (both positive) are parameters of supply function. These type of linear functions or relationship between different economic variable plays significant role in quantitative economics, which means that the market for a particular commodity can be represented by linear demand and supply functions. The equilibrium price is such that, at that price market demand equals the market supply and at that particular price, the market for that particular commodity clears. Let us assume equilibrium price to be Pe. Thus,

$$A - BPe = a + bPe$$

Solving for Pe, the study gets equilibrium price as $(A-a)/(B+b)$

Substituting the equilibrium price in demand function the study gets equilibrium quantity as:

$Qe = A\text{-}B(A\text{-}a)/(B+b) = [Ab+aB]/[B+b]$

In fact if all the parameter(a, b, A and B) are known,then the equilibrium price and quantity could be easily known.

If there is a shift in supply or demand function, as an example supply increases to become $S = á+bP$, where $á > a$, then the new equilibrium price and quantity would be :

$P'e = (A\text{-}á)/(B+b)$

$Q'e = (bA+Bá)/(B+b)$

Here, P'e is less then Pe and Q'e is greater than Qe. The study has computed this through the use of linear functions to show the relation in economic variables. It can be also seen in numbers if the parameters are made known. This is also true through economic intuition that if the supply of commodity increases then the equilibrium price Falls.

Comparative static analysis is one of the significant economic exercises which involves comparison of two different economic outcomes or equilibrium when one of the underlying parameter changes. For example, how does optimum output of a firm changes when the input price changes. How does the optimal bundle of a consumer changes when the price of a commodity changes. How does the optimum output of firm changes when the price of output changes. Comparative static analysis mathematically also involves use of multivariable calculus and matrix algebra.

Population Growth Analysis through Point Slope and Linear Equation

In UN report, the estimates of European population in 1960 was 641 million, and in 1970 the estimation was 705 million. Using these estimates, the study is now going to construct a linear function that will help us to provide the approximate population of Europe in the years to come.

Let 'P' denote the population in Millions, and the linear equation will be of the form $P = at+b$, where 't' denote the time in which $t=0$ is the initial year 1960 and $t = 10$ is the year 1970. This equation must pass through the points given by$(t_1, P_1) = (0,641)$ and $(t_2, P_2) = (10,705)$.

Here, the study will apply point slope formula for population forecastwhich is as follows:

$(y\text{-}y_1)/(x\text{-}x_1) = (y_2\text{-}y_1)/(x_2\text{-}x_1)$

Substituting the variables and the given data in the study,

(P-641)/(t-0)=(705-641)/(10-0)

On solving for 'P'the study gets, P=6.4t+641

Putting the value of 't'the study gives the estimates of European population through linear function.

Table 3

Year	1930	1975	2000
T	-30	15	40
United Nations Estimates *	573	728	854
Estimates of Point Slope Formula*	449	737	897

*All population estimates in millions; Source: Knut Sydsaeter, Peter Hammond with Arne Storm (2012)"Essential Mathematics for Economic Analysis

However, this point slope formula gives us estimates for economic analysis but the study can see that the figures obtained using this formula neither matchesnor goes nearby the figures provided by the UN estimates. This is so because population does not grow at a constant rate rather it grows with compound effect. To address this problem, the study will use another mathematical tool call exponential function that is population growth analysis through exponential functions.

According to UN estimates, European population was expected to grow by approximately 0.72 percent annually during the period 1960 to 2000.

Therefore population in 1961 would be P=641+(641*0.72)/100=641*1.0072 which approximates to 645 million.

Next year, in 1962 the population would be P=641*1.0072*(1+0.0072)=641*1.0072*1.0072

Which means that the population grows by the factor 1.0072 every year and in year 2000 the population will grow 40 times by the factor 1.0072

Conclusion

Through the use of econometric technique of regression analysis the study defines relationship between economic parameters of GDP and consumption. Also in comparative static analysis the studycompares different equilibrium outcomes using equations and mathematical operational tools in which equilibrium price falls and equilibrium quantity increases as an effect of increase in supply.

Another mathematical concept called compound effect or exponential function which is very important mathematical operation used in economic analysis of population growth. This tool helps us to forecast economic parameters in a better way because it takes us somewhere close to real or accurate figure.This is so because more population will have more children in future and the population will not grow at same or constant rate.

For advancement, mathematics has become the mode of communication in the analysis of economic models. It quantifies the relationship between economic variables and helps to identify the properties of these relationships.

Elementary economics use simple mathematical techniques to describe and focus models having properties of perfect competition, complete information and no other uncertainties that may exist in the real world business and economic environment. Whereas looking beyond introductory economics, intermediate aswell as advance micro economics and macroeconomics drop these text bookish assumptions. Hence, the mathematical demands to economic models become considerable high.

Some business analyst or managerial economist that applies mathematical tools and techniques to solve business related problems criticise mathematical modelling in economics.

Firstly, these mathematical models and techniques have no relation to the real world business problems or decisions that have to be made.

Secondly even if these models are able to make rare relation with the real world decision making then also there is not enough required data available on the relevant variables to apply these mathematical techniques.

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